



A further study of μ – τ symmetry breaking at neutrino telescopes after the Daya Bay and RENO measurements of θ_{13}

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ABSTRACT

Current neutrino oscillation data indicate that θ_{13} is not strongly suppressed and θ_{23} might have an appreciable deviation from $\pi/4$, implying that the 3×3 neutrino mixing matrix V does not have an exact μ – τ permutation symmetry. We make a further study of the effect of μ – τ symmetry breaking on the democratic flavor distribution of ultrahigh-energy (UHE) cosmic neutrinos at a neutrino telescope, and find that it is characterized by $|V_{\mu i}|^2 - |V_{\tau i}|^2$ which would vanish if either $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ or $\theta_{23} = \pi/4$ and $\delta = \pm\pi/2$ held. We observe that the second-order μ – τ symmetry breaking term $\bar{\Delta}$ may be numerically comparable with or even larger than the first-order term Δ in the flux ratios $\phi_e^T : \phi_\mu^T : \phi_\tau^T \simeq (1 - 2\Delta) : (1 + \Delta + \bar{\Delta}) : (1 + \Delta - \bar{\Delta})$, if $\sin(\theta_{23} - \pi/4)$ and $\cos \delta$ have the same sign. The detection of the UHE $\bar{\nu}_e$ flux via the Glashow-resonance channel $\bar{\nu}_e e \rightarrow W^- \rightarrow \text{anything}$ is also discussed by taking account of the first- and second-order μ – τ symmetry breaking effects.

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1. Introduction

Current experimental data have convinced us that three known neutrinos can oscillate from one flavor to another [1], implying the existence of a mismatch between their flavor and mass eigenstates. Hence the lepton flavors must mix as the quark flavors, and this phenomenon can be described by using an effective 3×3 unitary matrix V . In the basis where the flavor and mass eigenstates of three charged leptons are identical, V provides a unique link between the neutrino flavor (ν_e, ν_μ, ν_τ) and mass (ν_1, ν_2, ν_3) eigenstates:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \quad (1)$$

One may parametrize V in terms of three mixing angles and three phase angles as follows:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu, \quad (2)$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$ (for $ij = 12, 13, 23$), and $P_\nu = \text{Diag}\{e^{i\rho}, e^{i\sigma}, 1\}$ is physically relevant only if massive neutrinos are

the Majorana particles. It has been pointed out that $|V_{\mu i}| = |V_{\tau i}|$ (for $i = 1, 2, 3$) holds exactly, if either the conditions $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ or the conditions $\delta = \pm\pi/2$ and $\theta_{23} = \pi/4$ are satisfied [2]. However, this interesting μ – τ permutation symmetry must be broken: on the one hand, the fact that θ_{13} is not strongly suppressed has recently been established in the Daya Bay and RENO reactor antineutrino oscillation experiments [3]; on the other hand, the latest global fit of all the available neutrino oscillation data hints that the value of θ_{23} might have an appreciable deviation from $\pi/4$ [4]. An important task in today's experimental neutrino physics is therefore to determine the strength of μ – τ symmetry breaking, so as to help the theorists try different flavor symmetries and deeply understand the leptonic flavor mixing structure [5].

The neutrino telescopes (e.g., the running IceCube detector at the South Pole [6] and the proposed KM3NeT detector in the Mediterranean Sea [7]), which aim to observe the ultrahigh-energy (UHE) cosmic neutrinos and their flavor distributions, can serve as a novel probe of the μ – τ symmetry breaking effects. It is well known that neutrino oscillations may map $\phi_e^S : \phi_\mu^S : \phi_\tau^S = 1 : 2 : 0$, the initial flavor ratios of UHE cosmic neutrino fluxes produced from $p\gamma$ or pp collisions at a distant astrophysical source, into $\phi_e^T : \phi_\mu^T : \phi_\tau^T = 1 : 1 : 1$ at a neutrino telescope on the Earth [8] if there is the exact μ – τ permutation symmetry. It is also known that such a democratic flavor distribution can be broken down to [9]

$$\phi_e^T : \phi_\mu^T : \phi_\tau^T \simeq (1 - 2\Delta) : (1 + \Delta) : (1 + \Delta), \quad (3)$$

where

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$$\Delta = \frac{1}{2} \sin^2 2\theta_{12} \sin \varepsilon - \frac{1}{4} \sin 4\theta_{12} \sin \theta_{13} \cos \delta, \quad (4)$$

and $\varepsilon \equiv \theta_{23} - \pi/4$ for the parametrization of V given in Eq. (2). Hence Δ signifies the combined effects of μ - τ symmetry breaking, and whether its magnitude can reach the 10% level or not depends crucially on the sizes and signs of $\sin \varepsilon$ and $\cos \delta$. Note that the analytical approximation made in Eq. (3) does not reflect the difference between ϕ_μ^T and ϕ_τ^T , which should be given by the terms proportional to $\sin^2 \varepsilon$, $\sin^2 \theta_{13}$ and $\sin \varepsilon \sin \theta_{13}$ [10].

The purpose of this paper is to make a further study of μ - τ symmetry breaking relevant to the detection of UHE cosmic neutrinos originating from a certain cosmic accelerator. First, we calculate the flavor distribution of UHE cosmic neutrinos at a neutrino telescope in a parametrization-independent way. Such an exercise allows us to generalize the approximate result in Eq. (3) to the following exact one:

$$\phi_\alpha^T = \frac{\phi_0}{3} \left[1 + \sum_i |V_{\alpha i}|^2 (|V_{\mu i}|^2 - |V_{\tau i}|^2) \right] \quad (5)$$

for $\alpha = e, \mu$ and τ , where $\phi_0 = \phi_e^S + \phi_\mu^S + \phi_\tau^S$ is the total flux of UHE cosmic neutrinos and antineutrinos of all three flavors. Now it becomes transparent that a deviation from the democratic flavor distribution $\phi_e^T : \phi_\mu^T : \phi_\tau^T = 1 : 1 : 1$ at a neutrino telescope is characterized by the μ - τ symmetry breaking terms $|V_{\mu i}|^2 - |V_{\tau i}|^2$. In particular, we find that the difference between ϕ_μ^T and ϕ_τ^T is measured by $(|V_{\mu i}|^2 - |V_{\tau i}|^2)^2$. We also find that the second-order μ - τ symmetry breaking term $\bar{\Delta}$, which is a function of $\sin^2 \varepsilon$, $\sin^2 \theta_{13}$ and $\sin \varepsilon \sin \theta_{13}$, may be numerically comparable with or even larger than Δ in the realistic flux ratios $\phi_e^T : \phi_\mu^T : \phi_\tau^T \simeq (1 - 2\Delta) : (1 + \Delta + \bar{\Delta}) : (1 + \Delta - \bar{\Delta})$ if $\sin \varepsilon$ and $\cos \delta$ have the same sign. Second, we reexamine the effect of μ - τ symmetry breaking on the $\bar{\nu}_e$ flux of $E_{\bar{\nu}_e} \approx 6.3$ PeV which is detectable via the well-known Glashow-resonance (GR) channel $\bar{\nu}_e e \rightarrow W^- \rightarrow \text{anything}$ [11] at neutrino telescopes. Different from the previous result obtained Ref. [9], our new result for the GR-mediated $\bar{\nu}_e$ events will include the second-order μ - τ symmetry breaking effect.

2. Effects of μ - τ symmetry breaking

Let us define $\phi_\alpha^S \equiv \phi_{\nu_\alpha}^S + \phi_{\bar{\nu}_\alpha}^S$ and $\phi_\alpha^T \equiv \phi_{\nu_\alpha}^T + \phi_{\bar{\nu}_\alpha}^T$ (for $\alpha = e, \mu, \tau$) throughout this Letter, where $\phi_{\nu_\alpha}^S$ (or $\phi_{\bar{\nu}_\alpha}^S$) and $\phi_{\nu_\alpha}^T$ (or $\phi_{\bar{\nu}_\alpha}^T$) denote the ν_α and $\bar{\nu}_\alpha$ fluxes at a distant astrophysical source (or at a neutrino telescope), respectively. For most of the currently-envisaged sources of UHE cosmic neutrinos [12], a general expectation is that the initial neutrino fluxes are produced via the decay chain of charged pions and muons created from pp or $p\gamma$ collisions and their flavor content can be expressed as

$$\{\phi_e^S, \phi_\mu^S, \phi_\tau^S\} = \left\{ \frac{1}{3}, \frac{2}{3}, 0 \right\} \phi_0, \quad (6)$$

where $\phi_\tau^S = \phi_{\nu_\tau}^S = \phi_{\bar{\nu}_\tau}^S = 0$, and $\phi_0 = \phi_e^S + \phi_\mu^S + \phi_\tau^S$ is the total flux of neutrinos and antineutrinos of all flavors. Thanks to neutrino oscillations, the flavor distribution of such UHE cosmic neutrinos at a neutrino telescope is described by

$$\begin{aligned} \phi_\alpha^T &= \phi_{\nu_\alpha}^T + \phi_{\bar{\nu}_\alpha}^T = \sum_\beta [\phi_{\nu_\beta}^S P(\nu_\beta \rightarrow \nu_\alpha) + \phi_{\bar{\nu}_\beta}^S P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)] \\ &= \sum_i \sum_\beta (|V_{\alpha i}|^2 |V_{\beta i}|^2 \phi_\beta^S), \end{aligned} \quad (7)$$

where we have used

$$P(\nu_\beta \rightarrow \nu_\alpha) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha) = \sum_i |V_{\alpha i}|^2 |V_{\beta i}|^2. \quad (8)$$

Since the Galactic distance that the UHE cosmic neutrinos travel far exceeds the observed neutrino oscillation lengths, $P(\nu_\beta \rightarrow \nu_\alpha)$ and $P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$ are actually averaged over many oscillations and thus become energy- and distance-independent. Combining Eq. (7) with Eq. (6) and using the unitarity conditions of V , we find

$$\begin{aligned} \phi_\alpha^T &= \frac{\phi_0}{3} \sum_i |V_{\alpha i}|^2 (|V_{ei}|^2 + 2|V_{\mu i}|^2) \\ &= \frac{\phi_0}{3} \sum_i |V_{\alpha i}|^2 (1 + |V_{\mu i}|^2 - |V_{\tau i}|^2) \\ &= \frac{\phi_0}{3} \left[1 + \sum_i |V_{\alpha i}|^2 (|V_{\mu i}|^2 - |V_{\tau i}|^2) \right]. \end{aligned} \quad (9)$$

This is a simple proof of Eq. (5). Of course, the relationship $\phi_e^T + \phi_\mu^T + \phi_\tau^T = \phi_0$ holds. To be more explicit, we have

$$\begin{aligned} \phi_e^T &= \frac{\phi_0}{3} \left[1 + \sum_i |V_{ei}|^2 (|V_{\mu i}|^2 - |V_{\tau i}|^2) \right], \\ \phi_\mu^T &= \frac{\phi_0}{3} \left[1 + \sum_i |V_{\mu i}|^2 (|V_{\mu i}|^2 - |V_{\tau i}|^2) \right], \\ \phi_\tau^T &= \frac{\phi_0}{3} \left[1 + \sum_i |V_{\tau i}|^2 (|V_{\mu i}|^2 - |V_{\tau i}|^2) \right], \end{aligned} \quad (10)$$

from which we obtain the difference between ϕ_μ^T and ϕ_τ^T as

$$\phi_\mu^T - \phi_\tau^T = \frac{\phi_0}{3} \sum_i (|V_{\mu i}|^2 - |V_{\tau i}|^2)^2. \quad (11)$$

Now it becomes quite transparent that the deviation of ϕ_α^T (for $\alpha = e, \mu, \tau$) from $\phi_0/3$ is measured by $|V_{\mu i}|^2 - |V_{\tau i}|^2$, and the difference between ϕ_μ^T and ϕ_τ^T is purely an effect governed by $(|V_{\mu i}|^2 - |V_{\tau i}|^2)^2$. This exact and parametrization-independent observation is therefore useful for us to probe the leptonic flavor mixing structure via the detection of UHE cosmic neutrinos at neutrino telescopes.

Considering that the μ - τ symmetry of V is possible to be partly or softly broken, we define the following three μ - τ symmetry breaking quantities and express them in terms of three neutrino mixing angles and the Dirac CP-violating phase in the standard parametrization of V as given in Eq. (2):

$$\begin{aligned} \Delta_1 &\equiv |V_{\mu 1}|^2 - |V_{\tau 1}|^2 = (\sin^2 \theta_{12} - \cos^2 \theta_{12} \sin^2 \theta_{13}) \cos 2\theta_{23} \\ &\quad + \sin 2\theta_{12} \sin 2\theta_{23} \sin \theta_{13} \cos \delta, \\ \Delta_2 &\equiv |V_{\mu 2}|^2 - |V_{\tau 2}|^2 = (\cos^2 \theta_{12} - \sin^2 \theta_{12} \sin^2 \theta_{13}) \cos 2\theta_{23} \\ &\quad - \sin 2\theta_{12} \sin 2\theta_{23} \sin \theta_{13} \cos \delta, \\ \Delta_3 &\equiv |V_{\mu 3}|^2 - |V_{\tau 3}|^2 = -\cos^2 \theta_{13} \cos 2\theta_{23}. \end{aligned} \quad (12)$$

We see that the relationship $\Delta_1 + \Delta_2 + \Delta_3 = 0$ holds exactly, as guaranteed by the unitarity of V . There are three special but interesting cases, in which the μ - τ permutation symmetry is not completely broken:

- Case (A): $\theta_{23} \rightarrow \pi/4$. In this limit, Δ_i (for $i = 1, 2, 3$) can be simplified to

$$\begin{aligned} \Delta_1 &= -\Delta_2 = \sin 2\theta_{12} \sin \theta_{13} \cos \delta, \\ \Delta_3 &= 0. \end{aligned} \quad (13)$$

- Case (B): $\theta_{13} \rightarrow 0$. In this limit, the results in Eq. (12) are simplified to

$$\begin{aligned}\Delta_1 &= \tan^2 \theta_{12} \Delta_2 = \sin^2 \theta_{12} \cos 2\theta_{23}, \\ \Delta_3 &= -\cos 2\theta_{23}.\end{aligned}\quad (14)$$

- Case (C): $\delta \rightarrow \pm\pi/2$. In this limit, we simply obtain

$$\begin{aligned}\Delta_1 &= (\sin^2 \theta_{12} - \cos^2 \theta_{12} \sin^2 \theta_{13}) \cos 2\theta_{23}, \\ \Delta_2 &= (\cos^2 \theta_{12} - \sin^2 \theta_{12} \sin^2 \theta_{13}) \cos 2\theta_{23}, \\ \Delta_3 &= -\cos^2 \theta_{13} \cos 2\theta_{23}.\end{aligned}\quad (15)$$

Current experimental data tell us that case (A) is probably not true, case (B) is definitely not true, and case (C) remains an interesting possibility. In any case, we may make analytical approximations for ϕ_α^T (for $\alpha = e, \mu, \tau$) up to the accuracy of $\sin^2 \varepsilon$, $\sin^2 \theta_{13}$ and $\sin \varepsilon \sin \theta_{13}$. Then a result similar to Eq. (3) is

$$\phi_e^T : \phi_\mu^T : \phi_\tau^T \simeq (1 - 2\Delta) : (1 + \Delta + \bar{\Delta}) : (1 + \Delta - \bar{\Delta}), \quad (16)$$

where Δ has been given in Eq. (4), and $\bar{\Delta}$ is defined as¹

$$\begin{aligned}\bar{\Delta} &= (4 - \sin^2 2\theta_{12}) \sin^2 \varepsilon + \sin^2 2\theta_{12} \sin^2 \theta_{13} \cos^2 \delta \\ &\quad + \sin 4\theta_{12} \sin \varepsilon \sin \theta_{13} \cos \delta.\end{aligned}\quad (17)$$

It is easy to show that $\bar{\Delta} \geq 0$ holds for arbitrary values of δ , because the above expression can simply be transformed into $\bar{\Delta} = 3 \sin^2 \varepsilon + (\cos 2\theta_{12} \sin \varepsilon + \sin 2\theta_{12} \sin \theta_{13} \cos \delta)^2$, which may vanish if both $\sin \varepsilon = 0$ and $\sin \theta_{13} = 0$ (or $\cos \delta = 0$) hold.

One may define three working observables at neutrino telescopes [13] and link them to the μ - τ symmetry breaking quantities:

$$\begin{aligned}R_e &\equiv \frac{\phi_e^T}{\phi_\mu^T + \phi_\tau^T} = \frac{1 + \sum_i |V_{ei}|^2 \Delta_i}{2 - \sum_i |V_{ei}|^2 \Delta_i} \\ &\simeq \frac{1}{2} \left(1 + \frac{3}{2} \sum_i |V_{ei}|^2 \Delta_i \right) \simeq \frac{1}{2} - \frac{3}{2} \Delta, \\ R_\mu &\equiv \frac{\phi_\mu^T}{\phi_\tau^T + \phi_e^T} = \frac{1 + \sum_i |V_{\mu i}|^2 \Delta_i}{2 - \sum_i |V_{\mu i}|^2 \Delta_i} \\ &\simeq \frac{1}{2} \left(1 + \frac{3}{2} \sum_i |V_{\mu i}|^2 \Delta_i \right) \simeq \frac{1}{2} + \frac{3}{4} (\Delta + \bar{\Delta}), \\ R_\tau &\equiv \frac{\phi_\tau^T}{\phi_e^T + \phi_\mu^T} = \frac{1 + \sum_i |V_{\tau i}|^2 \Delta_i}{2 - \sum_i |V_{\tau i}|^2 \Delta_i} \\ &\simeq \frac{1}{2} \left(1 + \frac{3}{2} \sum_i |V_{\tau i}|^2 \Delta_i \right) \simeq \frac{1}{2} + \frac{3}{4} (\Delta - \bar{\Delta}).\end{aligned}\quad (18)$$

Comparing Eq. (11) and Eqs. (16)–(18), we immediately obtain

$$\begin{aligned}\phi_\mu^T - \phi_\tau^T &= \frac{\phi_0}{3} \sum_i \Delta_i^2 \simeq \frac{2\phi_0}{3} \bar{\Delta}, \\ R_\mu - R_\tau &\simeq \frac{3}{4} \sum_i \Delta_i^2 \simeq \frac{3}{2} \bar{\Delta}.\end{aligned}\quad (19)$$

Hence $\bar{\Delta}$ signifies the second-order effect of μ - τ symmetry breaking, and the departure of R_α (for $\alpha = e, \mu, \tau$) from $1/2$ is a clear measure of the overall μ - τ symmetry breaking effects.

Table 1

Possible sizes of μ - τ symmetry breaking at neutrino telescopes. The values of θ_{12} , θ_{13} , θ_{23} and δ are taken from the latest global analysis of currently available neutrino oscillation data done by Fogli et al. [4], where both normal and inverted neutrino mass hierarchies are considered.

Normal hierarchy	Best fit	2 σ range
$\sin^2 \theta_{12}$	3.07×10^{-1}	$(2.75 \dots 3.42) \times 10^{-1}$
$\sin^2 \theta_{13}$	2.45×10^{-2}	$(1.81 \dots 3.11) \times 10^{-2}$
$\sin^2 \theta_{23}$	3.98×10^{-1}	$(3.50 \dots 4.75) \times 10^{-1}$
δ	$0.89 \times \pi$	$0 \dots 2\pi$
Δ_1	-7.38×10^{-2}	$-0.151 \dots +0.256$
Δ_2	$+2.73 \times 10^{-1}$	$-0.135 \dots +0.365$
Δ_3	-1.99×10^{-1}	$-0.295 \dots -0.048$
Δ	-1.74×10^{-2}	$-0.096 \dots +0.026$
$\bar{\Delta}$	$+6.23 \times 10^{-2}$	$0 \dots 0.120$
Inverted hierarchy	Best fit	2 σ range
$\sin^2 \theta_{12}$	3.07×10^{-1}	$(2.75 \dots 3.42) \times 10^{-1}$
$\sin^2 \theta_{13}$	2.46×10^{-2}	$(1.83 \dots 3.13) \times 10^{-2}$
$\sin^2 \theta_{23}$	4.08×10^{-1}	$(3.55 \dots 6.27) \times 10^{-1}$
δ	$0.90 \times \pi$	$0 \dots 2\pi$
Δ_1	-8.19×10^{-2}	$-0.244 \dots +0.254$
Δ_2	$+2.61 \times 10^{-1}$	$-0.335 \dots +0.359$
Δ_3	-1.79×10^{-1}	$-0.285 \dots +0.249$
Δ	-1.28×10^{-2}	$-0.094 \dots +0.087$
$\bar{\Delta}$	$+5.56 \times 10^{-2}$	$0 \dots 0.115$

To illustrate the possible size of μ - τ symmetry breaking, we estimate Δ_i (for $i = 1, 2, 3$), Δ and $\bar{\Delta}$ by using the latest values of θ_{12} , θ_{13} , θ_{23} and δ obtained from a global analysis of the presently available neutrino oscillation data done by Fogli et al. [4]. Our numerical results are listed in Table 1, where both normal and inverted neutrino mass hierarchies are taken into account in accordance with Ref. [4]. Three comments are in order.

- $|\Delta_i| \sim 0.1$ (for $i = 1, 2, 3$) holds when the best-fit values of θ_{12} , θ_{13} , θ_{23} and δ are used. Given the 2σ intervals of the four input parameters, $|\Delta_i| \sim 0$ is allowed except for Δ_3 in the case of a normal neutrino mass hierarchy. It is therefore desirable to determine the departure of θ_{23} from $\pi/4$ and that of δ from $\pm\pi/2$ in the future experiments.
- Since the best-fit values of θ_{23} and δ lie in the ranges $0 < \theta_{23} < \pi/4$ and $\pi/2 < \delta < \pi$ respectively, $\sin \varepsilon$ and $\cos \delta$ have the same sign and thus the two terms of Δ in Eq. (3) significantly cancel each other. This significant cancellation leads to $|\Delta| < \bar{\Delta}$ at the percent level, implying a fairly good flavor democracy for ϕ_e^T , ϕ_μ^T and ϕ_τ^T .
- When the 2σ intervals of θ_{12} , θ_{13} , θ_{23} and δ are taken into account, the lower bound of Δ and the upper bound of $\bar{\Delta}$ are about 0.1 in magnitude. Hence the flavor democracy of ϕ_e^T , ϕ_μ^T and ϕ_τ^T can maximally be broken at the same level. Needless to say, an appreciable departure of R_α (for $\alpha = e, \mu, \tau$) from $1/2$ requires an appreciable effect of μ - τ symmetry breaking.

The rapid development of reactor antineutrino oscillation experiments implies that θ_{13} will soon become the best known angle of V . In this case, improving the precision of θ_{23} and determining the value of δ turn out to be two burning issues which will allow us to pin down the leptonic flavor mixing structure including CP violation.

3. On the Glashow resonance

Let us proceed to look at the effect of μ - τ symmetry breaking on the flavor distribution of UHE cosmic neutrinos at a neutrino

¹ This second-order perturbation term is consistent with the one obtained in Ref. [10], where the definition of ε has an opposite sign.

telescope by detecting the $\bar{\nu}_e$ flux from a distant astrophysical source through the GR channel $\bar{\nu}_e e \rightarrow W^- \rightarrow \text{anything}$ [11]. This reaction can take place over a narrow energy interval around the $\bar{\nu}_e$ energy $E_{\bar{\nu}_e}^{\text{GR}} \approx M_W^2/2m_e \approx 6.3$ PeV, and its cross sections of $\bar{\nu}_e N$ interactions of the same $\bar{\nu}_e$ energy [14]. A measurement of the GR reaction is important in neutrino astronomy because it may serve as a sensitive discriminator of UHE cosmic neutrinos originating from $p\gamma$ and pp collisions [15–18]. We hope that a neutrino telescope may measure both the GR-mediated $\bar{\nu}_e$ events and the $\nu_\mu + \bar{\nu}_\mu$ events of charged-current interactions in the vicinity of $E_{\bar{\nu}_e}^{\text{GR}}$, and their ratio can be related to the ratio of the $\bar{\nu}_e$ flux to the ν_μ and $\bar{\nu}_\mu$ fluxes entering the detector:

$$R_{\text{GR}} \equiv \frac{\phi_{\bar{\nu}_e}^{\text{T}}}{\phi_{\nu_\mu}^{\text{T}} + \phi_{\bar{\nu}_\mu}^{\text{T}}} = \frac{\phi_{\bar{\nu}_e}^{\text{T}}}{\phi_\mu^{\text{T}}}. \quad (20)$$

Here we follow Ref. [9] to reexamine the μ - τ symmetry breaking effect on R_{GR} .

The initial UHE cosmic neutrino fluxes are produced via the decay chain of charged pions and muons created from pp or $p\gamma$ collisions at a cosmic accelerator, and thus their flavor distribution can be expressed as

$$\begin{aligned} & \{\phi_{\nu_e}^{\text{S}}, \phi_{\bar{\nu}_e}^{\text{S}}, \phi_{\nu_\mu}^{\text{S}}, \phi_{\bar{\nu}_\mu}^{\text{S}}, \phi_{\nu_\tau}^{\text{S}}, \phi_{\bar{\nu}_\tau}^{\text{S}}\} \\ &= \begin{cases} \left\{ \frac{1}{6}, \frac{1}{6}, \frac{1}{3}, \frac{1}{3}, 0, 0 \right\} \phi_0 & (pp \text{ collisions}), \\ \left\{ \frac{1}{3}, 0, \frac{1}{3}, \frac{1}{3}, 0, 0 \right\} \phi_0 & (p\gamma \text{ collisions}). \end{cases} \end{aligned} \quad (21)$$

In either case the sum of $\phi_{\nu_\alpha}^{\text{S}}$ and $\phi_{\bar{\nu}_\alpha}^{\text{S}}$ (for $\alpha = e, \mu, \tau$) is consistent with ϕ_α^{S} in Eq. (6). Thanks to neutrino oscillations, the $\bar{\nu}_e$ flux at a neutrino telescope is given by

$$\phi_{\bar{\nu}_e}^{\text{T}} = \sum_\beta [\phi_{\bar{\nu}_\beta}^{\text{S}} P(\bar{\nu}_\beta \rightarrow \bar{\nu}_e)] = \sum_i \sum_\beta (|V_{ei}|^2 |V_{\beta i}|^2 \phi_{\bar{\nu}_\beta}^{\text{S}}). \quad (22)$$

To be explicit,

$$\begin{aligned} \phi_{\bar{\nu}_e}^{\text{T}}(pp) &= \frac{\phi_0}{6} \left(1 + \sum_i |V_{ei}|^2 \Delta_i \right) \simeq \frac{\phi_0}{6} (1 - 2\Delta), \\ \phi_{\bar{\nu}_e}^{\text{T}}(p\gamma) &= \frac{\phi_0}{3} \sum_i |V_{ei}|^2 |V_{\mu i}|^2 \\ &\simeq \frac{\phi_0}{12} [\sin^2 2\theta_{12} - 4\Delta + 2(1 + \cos^2 2\theta_{12}) \sin^2 \theta_{13}]. \end{aligned} \quad (23)$$

Since the expression of ϕ_μ^{T} can be found in Eq. (10), it is straightforward to calculate R_{GR} by using Eqs. (20) and (23) for two different astrophysical sources:

$$\begin{aligned} R_{\text{GR}}(pp) &\simeq \frac{1}{2} - \frac{3}{2}\Delta - \frac{1}{2}\bar{\Delta}, \\ R_{\text{GR}}(p\gamma) &\simeq \frac{\sin^2 2\theta_{12}}{4} - \frac{4 + \sin^2 2\theta_{12}}{4}\Delta - \frac{\sin^2 2\theta_{12}}{4}\bar{\Delta} \\ &\quad + \frac{1 + \cos^2 2\theta_{12}}{2} \sin^2 \theta_{13}. \end{aligned} \quad (24)$$

We see that the deviation of $R_{\text{GR}}(pp)$ from $1/2$ and that of $R_{\text{GR}}(p\gamma)$ from $\sin^2 2\theta_{12}/4$ are both controlled by the effects of μ - τ symmetry breaking, which can maximally be of $\mathcal{O}(0.1)$. As discussed in Refs. [15] and [17], the IceCube detector running at the South Pole has a good discovery potential to measure $R_{\text{GR}}(pp)$ after several years of data accumulation. In comparison, it seems more difficult to probe the GR-mediated UHE $\bar{\nu}_e$

events originating from the pure $p\gamma$ collisions at a cosmic accelerator.

Of course, one may also consider some other possible astrophysical sources of UHE cosmic neutrinos, such as the neutron beam source [19] with $\{\phi_e^{\text{S}} : \phi_\mu^{\text{S}} : \phi_\tau^{\text{S}}\} = \{1 : 0 : 0\}$ and the muon-damped source [20] with $\{\phi_e^{\text{S}} : \phi_\mu^{\text{S}} : \phi_\tau^{\text{S}}\} = \{0 : 1 : 0\}$, to study their flavor distributions at neutrino telescopes and probe the effects of μ - τ symmetry breaking.

4. Summary and further discussions

With the development of several neutrino telescope experiments, a lot of interest has recently been paid to the flavor issues of UHE cosmic neutrino fluxes and whether they are detectable in the foreseeable future [21]. In view of the fact that the smallest neutrino mixing angle θ_{13} is not strongly suppressed [3] and the hint that the largest neutrino mixing angle θ_{23} might have an appreciable departure from $\pi/4$ [4], we have carried out a further study of the effect of μ - τ symmetry breaking on the presumably democratic flavor distribution of UHE cosmic neutrinos at a neutrino telescope. Our new results are different from the previous ones in the following aspects:

- An exact and parametrization-independent expression for the fluxes of UHE cosmic neutrinos at a neutrino telescope is obtained in Eq. (5) or Eq. (10), and it clearly shows the μ - τ symmetry breaking effect measured by $|V_{\mu i}|^2 - |V_{\tau i}|^2$ (for $i = 1, 2, 3$). In addition, the difference between ϕ_μ^{T} and ϕ_τ^{T} is the pure second-order μ - τ symmetry breaking effect proportional to $(|V_{\mu i}|^2 - |V_{\tau i}|^2)^2$.
- The first- and second-order μ - τ symmetry breaking effects, characterized respectively by Δ and $\bar{\Delta}$ in the flux ratios $\phi_e^{\text{T}} : \phi_\mu^{\text{T}} : \phi_\tau^{\text{T}} \simeq (1 - 2\Delta) : (1 + \Delta + \bar{\Delta}) : (1 + \Delta - \bar{\Delta})$, may be numerically at the same order of magnitude. This observation is particularly true when $\sin \varepsilon$ and $\cos \delta$ have the same sign such that the two first-order μ - τ symmetry breaking terms of Δ cancel each other and then lead us to $|\Delta| \lesssim \bar{\Delta}$.
- $\bar{\Delta} \geq 0$ holds for arbitrary values of δ , and its contributions to the flux ratios $R_{\text{GR}}(pp)$ and $R_{\text{GR}}(p\gamma)$ for the GR-mediated $\bar{\nu}_e$ events are taken into account in Eq. (24). The term proportional to $\sin^2 \theta_{13}$ is also included in the expression of $R_{\text{GR}}(p\gamma)$, but such a term does not appear in $R_{\text{GR}}(pp)$.

Because of the poor data on ε and δ , whether the magnitude of Δ and $\bar{\Delta}$ (or one of them) can be as large as about 10% remains an open question. It is also possible that both of them are at the 1% level, as shown in TABLE I with the present best-fit results of θ_{23} and δ [4]. If $|\Delta|$ and $\bar{\Delta}$ are finally confirmed to be really small, then an approximately democratic flavor distribution $\phi_e^{\text{T}} : \phi_\mu^{\text{T}} : \phi_\tau^{\text{T}} \simeq 1 : 1 : 1$ will show up at neutrino telescopes for the UHE cosmic neutrinos originating from pp and (or) $p\gamma$ collisions at a distant astrophysical source.

So far we have omitted the uncertainties associated with the initial neutrino fluxes at a given astrophysical source. A more careful analysis of the flavor ratios of UHE cosmic neutrinos originating from pp or $p\gamma$ collisions yields $\phi_e^{\text{S}} : \phi_\mu^{\text{S}} : \phi_\tau^{\text{S}} \simeq 1 : 2(1 - \eta) : 0$ with $\eta \simeq 0.08$ [22]. If this uncertainty is taken into account, Eq. (5) or Eq. (9) will change to

$$\phi_\alpha^{\text{T}} \simeq \frac{\phi_0}{3} \left\{ 1 + \sum_i |V_{\alpha i}|^2 [(|V_{\mu i}|^2 - |V_{\tau i}|^2) - 2\eta |V_{\mu i}|^2] \right\} \quad (25)$$

for $\alpha = e, \mu$ and τ . This result clearly shows that the η -induced correction is in general comparable with (or even larger than) the effect of μ - τ symmetry breaking. On the other hand, there exist

the uncertainties associated with the identification of different flavors at the neutrino telescope. Taking the IceCube detector as an example, Beacom et al. [23] have pointed out that the experimental error for determining the ratio of the muon track to the non-muon shower is typically $\xi \sim 20\%$, depending on the event numbers. This estimate means that the ratio R_μ in Eq. (18) may in practice be contaminated by ξ , which is very likely to overwhelm the μ - τ symmetry breaking effect ($\Delta + \bar{\Delta}$).

We also admit that our treatment does not take account of the other complexities and uncertainties associated with the origin of UHE cosmic neutrinos, such as their energy dependence, the effect of magnetic fields and even possible new physics [24]. It is still too early to say that we have correctly understood the production mechanism of UHE cosmic rays and neutrinos from a given cosmic accelerator. But the progress made in the measurement of lepton flavor mixing parameters is quite encouraging, and it may finally allow us to well control the error bars from particle physics (e.g., the effect of μ - τ symmetry breaking) and thus concentrate on the unknowns from astrophysics (e.g., the initial flavor composition of UHE cosmic neutrinos). We believe that any constraint on the flavor distribution of UHE cosmic neutrinos to be achieved from a neutrino telescope will be greatly useful in diagnosing the astrophysical sources and in understanding the properties of neutrinos themselves. Much more efforts are therefore needed to make in this direction.

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